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## SOLUTIONS OF EXERCISES.

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### 355

REQUIRED the locus of the point in the normal to a conic, which is equally distant from the focus and the foot of the normal. [*Geo. R. Dean.*]

SOLUTION.

Taking the origin at the centre of the ellipse we have for the equation of the normal

$$y - b\sin\varphi = \frac{a}{b}\tan\varphi\,(x - a\cos\varphi)$$

or

$$y = \frac{a}{b}\tan\varphi\,.\,x - (a - b)\sin\varphi\,. \tag{1}$$

The locus required is the locus of the intersection of this normal with the line,

$$y = -\frac{a}{b}\tan\varphi\,.\,(x - ae)\,, \tag{2}$$

which passes through the focus. From (2)

$$\tan\varphi = -\frac{by}{a\,(x - ae)} \quad \text{and} \quad \sin\varphi = \frac{by}{\sqrt{b^2y^2 + a^2(x - ae)^2}}\,.$$

Eliminating $\varphi$ from (1) by means of these equations, we have, after reduction,

$$[(2x - ae)^2 - a^2e^4]\,(x - ae)^2 + (1 - e^2)\,y^2\,(2x - ae)^2 = 0\,,$$

a curve having $(2x - ae)^2 = 0$ for the equation of two coincident asymptotes. The curve cuts the axis of abscissæ at the points $x = \frac{1}{2}\,ae\,(1 \pm e)$, and the focus is a conjugate point. [*E. S. Crawley.*]

### 357

GIVEN $\tan_\kappa w = x + iy$, wherein

$$\tan_\kappa w = \chi\,(e^{w/\kappa} - e^{-w/\kappa})/(e^{w/\kappa} + e^{-w/\kappa})\,,$$

$$w = u + iv, \quad i = \sqrt{-1}\,,$$

and

$$\chi = m\,(\cos\beta + i\sin\beta)\,;$$

determine $u$ and $v$ as real quantities in terms of $x$, $y$, $m$, and $\beta$.

[*Irving Stringham.*]

## SOLUTION.

Let

$$\frac{e^{w/\kappa} - e^{-w/\kappa}}{e^{w/\kappa} + e^{-w/\kappa}} = \frac{x + iy}{m \cos \beta + im \sin \beta};$$

$$\therefore \quad e^{2w/\kappa} = \frac{m \cos \beta + x + i\,(m \sin \beta + y)}{m \cos \beta - x + i\,(m \cos \beta - y)} = \frac{s \,.\, e^{i\phi}}{t \,.\, e^{i\psi}} = \frac{s}{t}\, e^{i(\phi - \psi)},$$

in which $s$ is the tensor of the numerator and $t$ of the denominator; whence

$$w = \frac{1}{2}\,\kappa \left[\log_e \frac{s}{t} + (\varphi - \psi + 2h\pi)\, i\right],$$

in which $h$ is an arbitrary integer;

$$\therefore \quad u = \frac{1}{2}\, m \,.\, \cos \beta \,.\, \log \frac{s}{t} - \frac{1}{2}\, m \,.\, \sin \beta \,.\, (\varphi - \psi + 2h\pi),$$

$$v = \frac{1}{2}\, m \,.\, \cos \beta \,.\, (\varphi - \psi + 2h\pi) + \frac{1}{2}\, m \,.\, \sin \beta \,.\, \log \frac{s}{t};$$

in which

$$\log \frac{s}{t} = \frac{1}{2} \log \frac{m^2 + x^2 + y^2 + 2mx \cos \beta + 2my \sin \beta}{m^2 + x^2 + y^2 - 2mx \cos \beta - 2my \sin \beta},$$

$$\varphi = \tan^{-1} \frac{m \sin \beta + y}{m \cos \beta + x}, \qquad \psi = \tan^{-1} \frac{m \sin \beta - y}{m \cos \beta - x};$$

and, consequently,

$$\varphi - \psi = \tan^{-1} \frac{2m\,(y \cos \beta - x \sin \beta)}{m^2 - x^2 - y^2}.$$

If

$$r = \sqrt{x^2 + y^2}, \qquad \theta = \tan^{-1} \frac{y}{x},$$

$$\lg \frac{s}{t} = \frac{1}{2} \lg \left[\frac{m^2 + r^2 + 2mr \cos (\theta - \beta)}{m^2 + r^2 - 2mr \cos (\theta - \beta)}\right];$$

$$\varphi - \psi = \tan^{-1} \frac{2mr \sin (\theta - \beta)}{m^2 - r^2}.$$

[*J. B. Shaw, Jr.*]

## 358

A HEMISPHERICAL bowl, external and internal radii $R$ and $r$, has a sphere of the same material, radius $a$, suspended from the edge of the rim. Determine the inclination of the plane of the rim to the horizontal plane on which the bowl stands. [*Artemas Martin.*]

### SOLUTION.

The distance of the centre of gravity of the bowl from the centre of the rim is

$$\frac{\pi\,(R^4 - r^4)}{4M},$$

where $M$ is the volume of the bowl.

Let $a$ be the angle of inclination of the bowl to the horizon, and take moments about the point of support, the vertical through which passes through the centre of the rim, the line of action of the upward pressure being normal to the surface.

We get

$$\tan a = \frac{16Ra^3}{3\,(R^4 - r^4)}.$$

[*Geo. R. Dean.*]

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## EXERCISES.

### 366

Show that for any parabola $y = x^2 + ax + b$, the area included between the curve, any two ordinates, and the $x$-axis, is equal to the product of the ordinate midway between the bounding ordinates and the interval between them, plus one-twelfth the cube of this interval. [*W. H. Echols.*]

### 367

On the axis of $x$ take any point $x_m$, and equi-distant from it on either side select points $a$ and $b$. Draw verticals through $a$ and $b$, also the circle on $ab$ as diameter. With $a$ and $b$ as centers and radius equal to a side of the inscribed square draw arcs cutting the verticals at $a$ and $b$ in $a'$ and $b'$. The straight lines $a'x_m$ and $b'x_m$ cut the circle in points whose abscissæ are $x_1$ and $x_2$, respectively.

Show that if any cubic* of the form

$$y = x^3 + \alpha x^2 + \beta x + \gamma$$

be described passing through any two points $x_1y_1$ and $x_2y_2$ on the verticals at $a$ and $b$ respectively, then the area included between the curve, the verticals at $a$ and $b$, and the $x$-axis, is constant and equal to

$$\tfrac{1}{2}(y_1 + y_2)(b - a).$$

[*W. H. Echols.*]

### 368

A homogeneous ellipsoid of weight $W$ and semi-axes $a$, $b$, $c$ has a weight $P$ suspended from a point on the surface whose coordinates, referred to the axial planes, are $l$, $m$, $n$. Find the inclinations of the axial planes to the horizontal plane upon which the ellipsoid rests. [*H. Y. Benedict.*]

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* Of course the same applies to any curve of lower degree, as the parabola

$$y = x^2 + px + q.$$